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Persistent currents in annuli: effects of disorder and Coulomb interaction

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Abstract. We investigate the persistent current of spinless electrons in a mesoscopic annulus threaded by a magnetic flux. As in the tight-binding systems, the excitations in the radial direction generate additional discontinuities in the persistent current, which are rounded off by disorder. The Coulomb interaction, the effects of which are investigated within the Hartree–Fock approximation, is found to suppress the persistent current in the presence of weak disorder. For strong disorder corresponding to the diffusive regime, the persistent current is enhanced by weak interactions.

1. Introduction

Electrons in mesoscopic metal rings, threaded by a magnetic flux Φ, are known to carry circulating currents which are periodic in Φ with the periodicity of the flux quantum Φ_0 ($\equiv hc/e$) [1]. Such persistent currents were first observed in the experiment on 10^7 isolated copper rings [2]: the system of rings exhibited an ensemble-averaged current of magnitude $\approx 3\times 10^{-3} ev_F/L$ and periodicity $\Phi_0/2$, where L is the circumference of rings and v_F is the Fermi velocity of the electrons. Subsequent experiments on a single isolated gold ring [3] also displayed a persistent current with the period Φ_0. However, its magnitude was of the order of ev_F/L, which is much larger than the theoretical prediction in the diffusive regime [4], and has been a source of controversy. On the other hand, the experiment investigating the ballistic regime in the GaAs/GaAlAs system [5] yielded a current in good agreement with the theoretical value.

There have been attempts to attribute the origin of the discrepancy between experimental measurement and theoretical prediction to electron–electron interactions: although the first-order calculation in the diffusive regime yielded some enhancement of the disorder-averaged current in agreement with experiment, consideration of the multiple-scattering corrections in turn led to a strong reduction [6]. The semi-classical theory predicting the average current to be two orders of magnitude larger [7] was followed by serious questions as to the validity of the approximation employed [8]. Furthermore, strong enhancement of the typical current was also reported [9], but the inconsistency in the manner of evaluating diagrams was pointed out [10]. These controversial points in analytical studies have motivated numerical investigation of the interaction effects on persistent currents in disordered rings, which have been performed mostly within the tight-binding model. In 1D disordered rings, both the nearest-neighbour repulsion [11] and the Coulomb interaction [12] have been found to suppress the persistent currents of spinless electrons, whereas the repulsive Hubbard

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interaction between electrons with spin 1/2 can enhance the average currents depending on the configuration and the degree of disorder [13], which is supported by the first-order perturbation theory [14] as well as by the renormalization group calculations [15, 16]. Since the 1D ring with spin-1/2 electrons can be considered to have effectively two channels, such enhancement is also expected in higher dimensions. Recent investigation of multichannel systems has indeed shown signs of enhancement due to the Coulomb interaction [17]. In those studies employing the tight-binding model, however, the number of sites considered is very limited, and severe finite-size effects are unavoidable. This makes it desirable to study the interaction effects in continuum systems; still there have been few numerical studies even on the effects of disorder in two- or three-dimensional continuum systems, let alone the interaction effects. Recently the transfer matrix method was employed in the calculation of persistent currents on a finite-width mesoscopic ring of just two channels, in the presence of only two impurities of fixed positions and strengths [18]: there the diffusive regime with strong disorder as well as the electron–electron interactions were not considered.

In this paper, we study the persistent currents of the spinless electrons in two-dimensional (2D) mesoscopic annuli, laying emphasis on the effects of impurities and the Coulomb interaction. In the absence of electron–electron interactions, additional discontinuities are observed in the current–flux characteristics. Here disorder tends to round off these discontinuities and to suppress the current, which is similar to the case of the tight-binding model. The Coulomb interaction, which is introduced within the Hartree–Fock approximation, is found to suppress the current in the presence of weak disorder; in the diffusive regime, in contrast, the current is enhanced by the interaction unless it is too strong. The range of the interaction strength resulting in enhancement is found to grow with the strength of the disorder.

2. Noninteracting systems without disorder

We first consider N spinless free electrons on a mesoscopic annulus of inner radius R_1 and outer radius R_2 threaded by a magnetic flux Φ. Each electron, possessing mass m_e and charge $-e$, is described by the Schrödinger equation, which yields eigenenergies and eigenfunctions [18]

$$\begin{aligned}
E_{nm} &= \frac{\hbar^2}{2m_e}\left(\frac{\gamma_{nm}}{R_1}\right)^2 \\
\Psi_{nm}(r,\theta) &= \frac{\pi}{\sqrt{2}}\frac{\gamma_{nm}}{R_1}\left\{\left[\frac{N_{m+f}(\gamma_{nm})}{N_{m+f}(\mu\gamma_{nm})}\right]^2 - 1\right\}^{-1/2} \\
&\quad \times \left[N_{m+f}(\gamma_{nm})J_{m+f}(\gamma_{nm}r/R_1) - J_{m+f}(\gamma_{nm})N_{m+f}(\gamma_{nm}r/R_1)\right]\frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}m\theta} \\
&\equiv \psi_{nm}(r)\frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}m\theta} \qquad n = 1, 2, 3, \ldots; m = 0, \pm 1, \pm 2, \ldots.
\end{aligned} \tag{1}$$

Here $J_m(x)$ and $N_m(x)$ are the mth Bessel functions of the first kind and the second kind, respectively, and γ_{nm} represents the nth zero of the equation

$$J_{m+f}(x)N_{m+f}(\mu x) - N_{m+f}(x)J_{m+f}(\mu x) = 0$$

with $\mu \equiv R_2/R_1$ and $f \equiv \Phi/\Phi_0$. The eigenfunctions given by equation (1) then lead to the current density carried by an electron at level (n, m):

$$j_{nm}(r) \equiv -\frac{e}{m_e}\mathrm{Re}\left[\Psi^*_{nm}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\right)_\theta \Psi_{nm}\right] = -\frac{\hbar e}{2\pi m_e}(m+f)\frac{\psi^2_{nm}(r)}{r} \tag{2}$$

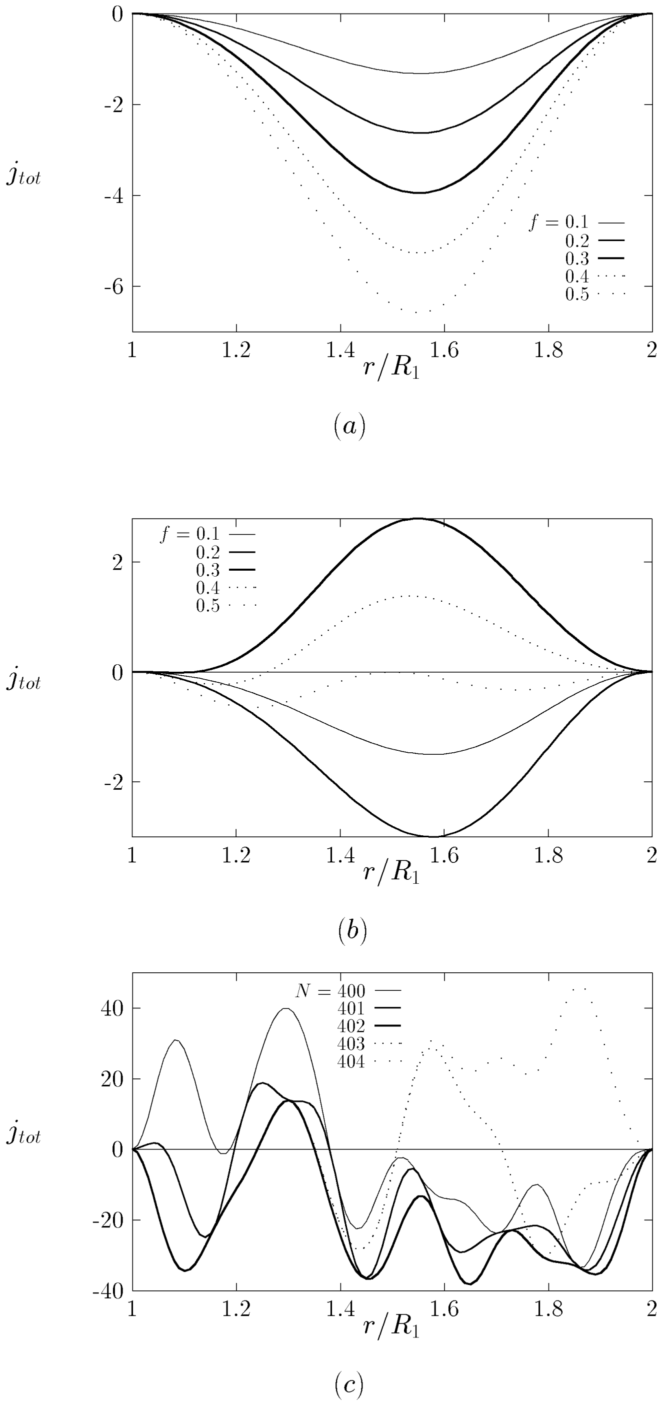


Figure 1. Persistent current density as a function of r in a clean annulus with $\mu = 2$ for several values of N and f: (*a*) $N = 15$; (*b*) $N = 17$; (*c*) $f = 0.2$. The current density j_{tot} is expressed in units of $e\hbar/2\pi m_e R_1^3$.

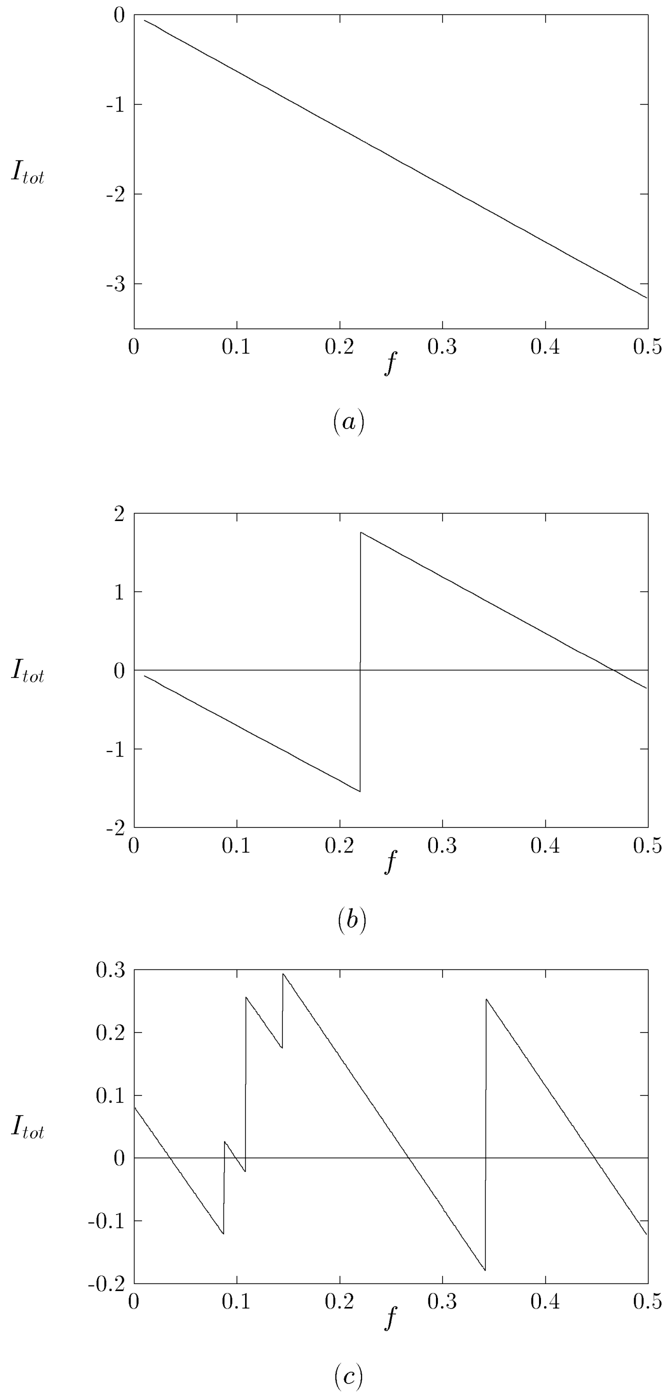


Figure 2. Total current (in units of $e\hbar/2\pi m_e R_1^2$) as a function of f in a clean annulus for (*a*) $N = 15$ and $\mu = 2$; (*b*) $N = 17$ and $\mu = 2$; (*c*) $N = 99$ and $\mu = 11$.

and the total current density is given by the sum of equation (4) over all occupied levels:

$$j_{tot}(r) = \sum_{n,m}{}' j_{nm}(r) = -\frac{\hbar e}{2\pi m_e} \sum_{n,m}{}' (m+f)\frac{\Psi_{nm}^2(r)}{r}. \tag{3}$$

Figure 1 shows the total current density (in units of $e\hbar/2\pi m_e R_1^3$) as a function of r for several values of N and f in an annulus of $\mu = 2$. For small N, the current flows in the same direction everywhere and the direction does not change with f, similarly to the case for 1D rings. As N is increased, however, the direction is no longer independent of f and may even change according to r (figure 1(*b*)). Furthermore, in a system with a large number of electrons, figure 1(*c*) shows that the behaviour of current density as a function of r is quite erratic and very sensitive to the number of electrons.

The total current flowing in the annulus is given by

$$I_{tot} = \int_{R_1}^{R_2} j_{tot}(r)\, \mathrm{d}r \tag{4}$$

which is displayed as a function of f in figure 2. It is obvious here that I_{tot} is a periodic odd function of f with period unity, allowing us to restrict the range of f to $[0, 1/2]$. Interestingly, the total current is remarkably linear in f, as in 1D rings. Unlike in 1D rings, however, there can appear discontinuities at values of f other than half-integers. These additional discontinuities in the 2D annulus are due to the existence of the excitations in the radial direction and have also been observed in the tight-binding multichannel systems [19, 20]. These discontinuities in general grow in number with the number of electrons, and become more pronounced in the ring with large μ, where the excitation energies in the radial direction are not so high compared with those in the angular direction (see figure 2(*c*)). There are no negative discontinuities in the total current as noted in [20].

3. Noninteracting systems with disorder

We next investigate the effects of disorder. For simplicity, we assume that a single impurity at random position $\boldsymbol{r}_i$ is characterized by the δ-function potential with random strength W_i, and include in the Hamiltonian the potential due to N_{imp} impurities:

$$W_{imp} = \sum_{i=1}^{N_{imp}} W_i\, \delta(\boldsymbol{r} - \boldsymbol{r}_i) \tag{5}$$

where W_i and $\boldsymbol{r}_i$ are quenched random variables distributed uniformly in the range $[-W_{max}/2, W_{max}/2]$ and on the whole annulus, respectively. For convenience we adopt the matrix representation of the Hamiltonian, and write the components in the basis consisting of the eigenstates of the clean system:

$$H_{ab} \equiv E_a^0 \delta_{ab} + \sum_{i=1}^{N_{imp}} W_i \Psi_a^*(\boldsymbol{r}_i)\Psi_b(\boldsymbol{r}_i) \tag{6}$$

where a and b label the eigenstates, i.e., $a \equiv (n, m)$, etc, in the previous notation, and E_a^0 is the energy of the eigenstate a in the clean system. Simple diagonalization of this matrix leads to the eigenstates in the presence of disorder, from which the total current can be computed.

We have investigated the systems with 10 electrons and 30 impurities. Figure 3 shows the average current as a function of f, calculated with 15 basis functions. The average over disorder, the strength of which is controlled by W_{max}, has been taken over 50–1000 configurations. To check whether the number of basis functions is sufficient, we have also increased the number to 50 in the calculation of the current, and obtained the same results within numerical accuracy. It is found that weak disorder rounds off discontinuities in the current–flux curve. In this case, the current at the flux corresponding to the discontinuity (in the absence of disorder) is given by the average of the two limit values at the discontinuities, which indicates that the disorder mixes the electron states near the Fermi surface. As the disorder strength is increased, the occupation probabilities of higher energy levels increase and the contributions from different levels tend to cancel out, resulting in suppression of the current. The first harmonics become dominant and the additional discontinuity disappears for strong disorder. It is of interest to note that the effects of impurities, which are distributed randomly on the annulus, are very similar to those of the on-site random energy in the tight-binding model [19, 20]. This is in contrast with [18], where a mesoscopic annulus with two channels and two impurities of fixed strengths and positions was considered via the transfer matrix method. There it was observed that the second harmonics are pronounced in the Fourier expansion of the persistent current in the ballistic regime, and the halving of the fundamental period Φ_0 has been attributed to the interchannel coupling due to impurity scattering. The above results, however, demonstrate that the first harmonics become more important with the strength of disorder, which excludes the possibility of the period halving arising from the interchannel coupling.

4. Interacting systems with disorder

In this section we consider the effects of the Coulomb interaction, and include in the Schrödinger equation the terms

$$U(\boldsymbol{r})\Psi_i(\boldsymbol{r}) + \sum_{j=1}^{N} \int \mathrm{d}^2\boldsymbol{r}' \; \frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|} \Psi_j^*(\boldsymbol{r}')[\Psi_j(\boldsymbol{r}')\Psi_i(\boldsymbol{r}) - \Psi_j(\boldsymbol{r})\Psi_i(\boldsymbol{r}')] \tag{7}$$

where the first term represents the interaction with the positive background charges distributed uniformly on the annulus of area A:

$$U(\boldsymbol{r}) \equiv -\frac{Ne^2}{A} \int \mathrm{d}^2\boldsymbol{R} \; \frac{1}{|\boldsymbol{r}-\boldsymbol{R}|}.$$

The electron–electron interactions are included within the Hartree–Fock approximation in the second term. The Hamiltonian matrix in the presence of both disorder and the Coulomb interaction then reads

$$H_{ab} = E_a^0\delta_{ab} + \sum_{i=1}^{N_{imp}} W_i \Psi_a^*(\boldsymbol{r}_i)\Psi_b(\boldsymbol{r}_i) + U_{ab} + \sum_{j=1}^{N}\sum_{c,d} C_{jc}^* C_{jd}(V_{acdb} - V_{acbd}) \tag{8}$$

where

$$U_{ab} \equiv \int \mathrm{d}^2\boldsymbol{r} \; \Psi_a^*(\boldsymbol{r})U(\boldsymbol{r})\Psi_b(\boldsymbol{r})$$

$$V_{abcd} \equiv \int \mathrm{d}^2\boldsymbol{r} \int \mathrm{d}^2\boldsymbol{r}' \; \Psi_a^*(\boldsymbol{r})\Psi_b^*(\boldsymbol{r}')\frac{e^2}{|\boldsymbol{r}-\boldsymbol{r}'|}\Psi_c(\boldsymbol{r}')\Psi_d(\boldsymbol{r})$$

and the C_{ja} are the expansion coefficients defined by

$$\Psi_j(\boldsymbol{r}) \equiv \sum_a C_{ja}\Psi_a(\boldsymbol{r}).$$

The coefficients can be obtained by means of the iterated diagonalization of the Hamiltonian matrix.

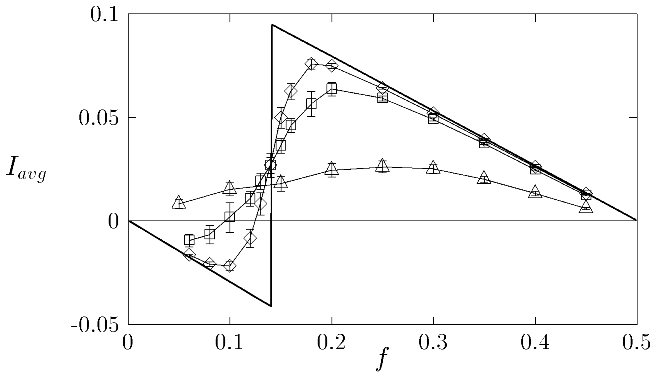


Figure 3. Average current as a function of f in a disordered annulus with $N = 10$ and $N_{imp} = 30$. The data marked by diamonds (◊), squares (□), and triangles (△) correspond to the disorder strength $W \equiv (m/\pi\hbar^2)W_{max} = 0.1$, 0.2, and 0.6, respectively. The thick line represents the current in the clean system ($W = 0$).

We have investigated seven interacting electrons scattered by 30 impurities, using up to 35 basis functions and 500 disorder configurations, and present in figure 4 the resulting current–flux characteristics in systems (*a*) with weak disorder and (*b*) with strong disorder, which display an additional discontinuity at $f \approx 0.40$ in the clean limit. We first concentrate on the region far from this discontinuity. Figure 4(*a*) shows that for weak disorder the current is in general reduced by the interaction. In a system with strong disorder, on the other hand, weak interactions enhance the current, whereas stronger interactions again tend to suppress the current. (Compare the cases $V \equiv 2me^2R_1/\hbar^2 = 0.5$ and 1.0 with the noninteracting case ($V = 0$) in figure 4(*b*).) These behaviours can be clearly observed in figure 5(*a*), where the currents for $f = 0.25$ and several disorder strengths are displayed as functions of the interaction strength. In addition, the strength of the interaction, which generates the maximum current, is shown to increase with the disorder strength. Here each point has been obtained after averaging over 1500 disorder configurations. The dependence of the current on the disorder strength has also been examined and displayed in figure 5(*b*), which shows that the current always decreases monotonically with disorder, regardless of the interaction strength. These observations lead to the general conclusion that the interaction always reduces the current in the ballistic regime, like in the clean system. This is to be contrasted with the case for the diffusive regime, where weak interactions tend to screen the impurity potential, enhancing the current. For strong interactions, on the other hand, correlations between electrons become important, leading to suppression of the current.

Remarkably, figure 4(*a*) displays quite a different behaviour near $f \approx 0.40$, where the discontinuity appears in the clean limit. Here the cooperative effects of the interaction and disorder enhance the current drastically even in the ballistic regime, apparently removing the discontinuity. The discontinuity in the noninteracting system appears due to the level-crossing of the two states $(n, m) = (1, 3)$ and $(2, 0)$; the latter carries a negative current much smaller than that of the former. In the presence of weak disorder, these two states are mixed near the discontinuity, while the level $(2, 0)$ is shifted upwards due to the Coulomb interaction with the filled level $(1, 0)$. These two effects effectively eliminate the level-crossing of the two states, yielding figure 4(a).

The typical current, which is defined by the root mean square of the total current, can also serve as a measure of the magnitude of the persistent current. We have thus computed

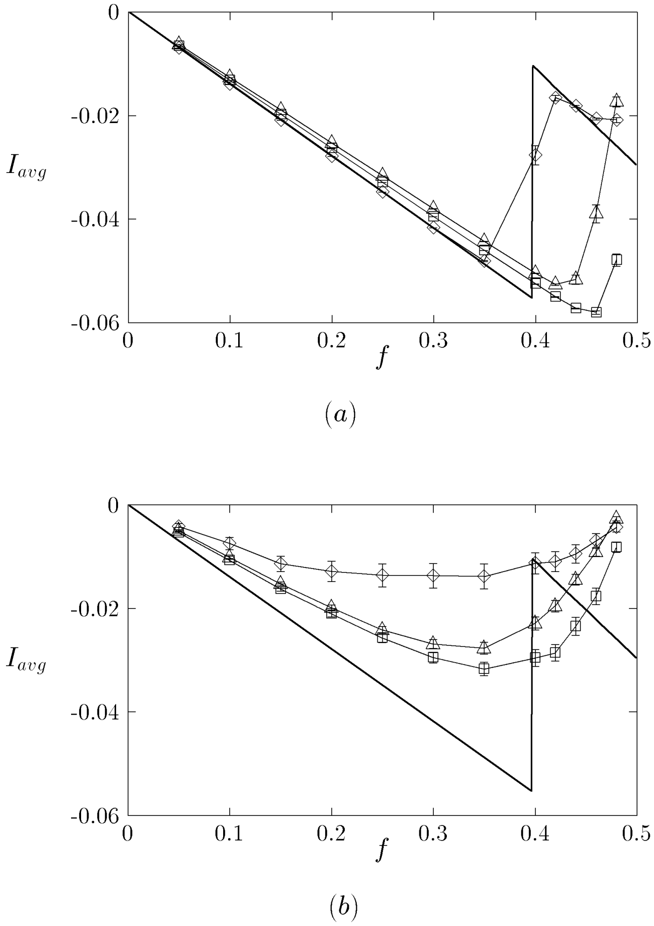
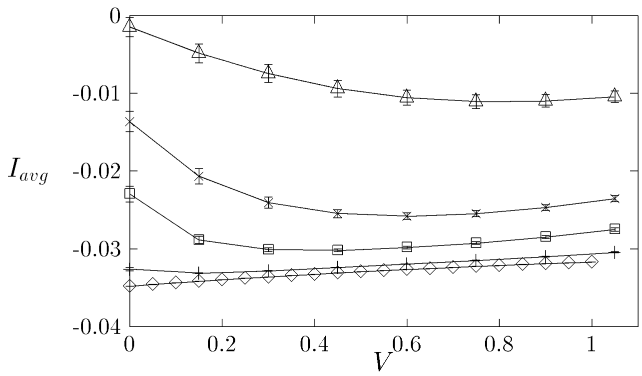


Figure 4. Average current as a function of f in the presence of both disorder and the Coulomb interaction, for (*a*) $W = 0.02$ and (*b*) $W = 0.3$. The data marked by diamonds, squares, and triangles correspond to the interaction strengths $V = 0$, 0.5, and 1.0, respectively. The thick lines represent the current in the noninteracting clean system ($V = W = 0$).

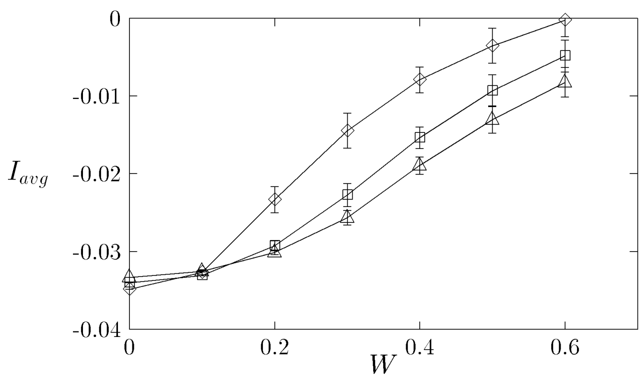
the typical currents in the system with the same parameter values. Figure 6 displays the typical current as a function of f, the behaviour of which is qualitatively similar to that of the average current: the Coulomb interaction in general suppresses the typical current in the weak-disorder regime whereas for strong disorder enhancement is caused by weak interactions. The dependence of the typical current on the interaction strength, which is presented in figure 7, also exhibits features largely similar to those of the average current. Thus the above conclusions are in general applicable to the typical current as well as the average current of the system.

5. Conclusion

We have investigated the persistent current in an annulus threaded by a magnetic flux, with emphasis on the effects of impurities and electron–electron interactions. It has been observed that the persistent current in such a continuum system also displays additional



(a)



(b)

Figure 5. (*a*) Average current as a function of V for several disorder strengths. The data marked by diamonds, plus signs, squares, crosses, and triangles correspond to $W = 0.02$, 0.1, 0.2, 0.3, and 0.6, respectively. (*b*) Average current as a function of W for several interaction strengths. The data marked by diamonds, squares, and triangles correspond to $V = 0.0$, 0.2, and 0.4, respectively.

discontinuities. They are rounded off by weak disorder, which mixes the states near the Fermi level. Here strong disorder leads to substantial suppression of the average current, in which the first harmonics are dominant. The Coulomb interaction has been found to play two different roles according to the disorder strength. In the presence of weak disorder, it always suppresses both the average current and the typical current, whereas in the diffusive regime with strong disorder both are enhanced by weak interactions. A similar trend in the behaviour of the average current was also observed in 1D systems: in the renormalization group calculation of the 1D ring with the repulsive Hubbard interaction [16] as well as in the Monte Carlo simulations of the 1D spinless Luttinger liquid [21]. However, no such results are available on the typical current, even in one dimension.

Finally, we point out some peculiar features of persistent currents obtained in this work compared with those in the existing works. First, it should be remarked that the sign of the average current can be either paramagnetic or diamagnetic, in contrast with the paramagnetic

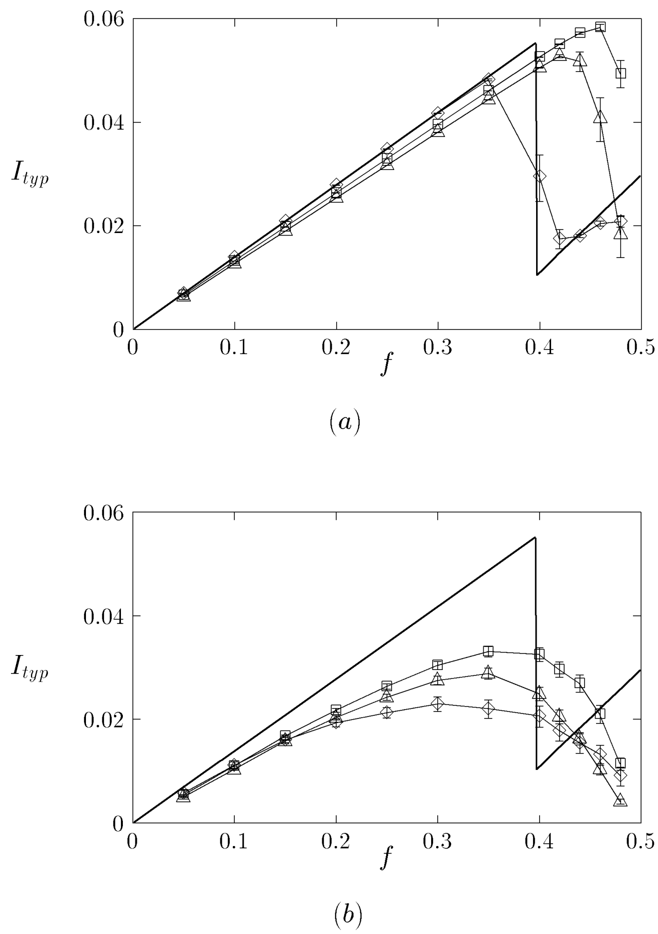


Figure 6. Typical current as a function of f in the presence of both disorder and the Coulomb interaction, for (*a*) $W = 0.02$ and (*b*) $W = 0.3$. The symbols are the same as in figure 4.

behaviour in the case of the diffusive electrons in a quasi-1D ring [6, 7, 8]. In a clean system, the sign is determined by the number of radial excitations below the Fermi level as well as the number of electrons. In disordered systems, on the other hand, the sign is also affected by the contributions of higher-energy levels which may carry currents of the opposite sign, although the interactions in general do not change the sign. It is also worthwhile to mention the apparent enhancement of the typical current in the diffusive regime, which is to be compared with the insensitivity of the typical current in a quasi-1D ring [10]. Unlike the case for the average current, however, the enhancement of the typical current becomes weaker as the disorder strength is increased further, and the possibility of no enhancement of the typical current in the strong-disorder limit cannot be excluded. Lastly, it is noted that disorder always suppresses the average current regardless of the interaction strength—and thus even in a noninteracting system. This feature can also be observed in the analytical results [22] as well as in the numerical work on multichannel systems [17].

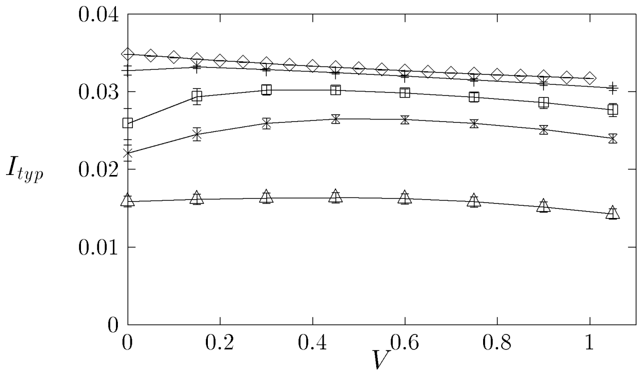


Figure 7. Typical current as a function of V for several disorder strengths. The same symbols are used as in figure 5(a).

Acknowledgments

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